Stat 455 Lecture 10: Test in Non-Gaussian Mixed Models (09-27-2007)

1 Introduction

For test in non-Gaussian linear mixed model, we introduce empirical method of moments (EMM) for estimating kurtoses of random effects (αs) and random errors (ϵs), which are required for adjusting the asymptotic covariance-variance matrix for test statistic. Since the test is based on asymptotic normal theory.

2 Asymptotic Normal Distribution and Hypothesis Test

Textbook p56.

3 Empirical Method of Moments (EMM)

- ◇ Motivation: except for estimating a vector of parameters (say β), we also want to evaluate the estimation precision using additional parameters (say σ^2, which also need to be estimated). EMM does this.
- ◇ Advantages of EMM:
  1. Simple, often results in closed-form expressions
  2. Relies on weak distributional assumptions.
  3. Applied to various fields.
- ◇ Outline of EMM:
  1. θ is a vector of parameters with consistent estimator $\hat{\theta}$; ϕ is a vector of additional parameters about which knowledge is needed. Denote $M(\psi, y) = M(\theta, \phi, y)$ is a vector of functions with same dimension as ϕ, y is observation.
  2. Suppose $E_Y(M(\theta, \phi, y)) = 0$. If θ is known, we solve
  $$M(\theta, \phi, y) = 0 \tag{1}$$
  for ϕ.
  3. Given consistent estimator $\hat{\theta}$, we work on
  $$M(\hat{\theta}, \phi, y) = 0 \tag{2}$$
  for ϕ.

4 Example:

$$M(\theta,\phi,y) = \partial l(\theta,\phi;y)/\partial\phi$$

and

$$\partial l(\hat{\theta},\phi;y)/\partial\phi = 0.$$

- **Theorem 1** in Jiang (2002).
- EMM Examples:

1 Generalized Linear Models in Jiang (2002).

2 ARMA Models in Jiang (2002).

3 Simple Random Effects Models in Jiang (2002).

4 Application: One-way Random Effects Model

We use a simple random effects model to demonstrate the EMM.

4.1 Theoretical Asymptotic Variance of REML Estimator

For

$$y_{ij} = \beta + \alpha_i + \epsilon_{ij}, (i = 1, ...m, j = 1, ..., n) \tag{3}$$

where, Hartley-Rao (1967) notation: $\lambda = \sigma_\epsilon^2$ and $\gamma = \sigma_\alpha^2/\sigma_\epsilon^2$.
Theoretical asymptotic variances of REML for λ and γ (Jiang, 1998):

1) $\sqrt{mn}(\hat{\lambda} - \lambda)$ is $\lambda^2(2 + k_0)$

2) $\sqrt{mn}(\hat{\gamma} - \gamma)$ is $\gamma^2(2 + k_1)$

3) $k_0 = E(\epsilon_{11}^4)/\sigma_0^4 - 3$

4) $k_1 = E(\alpha_1^4)/\sigma_1^4 - 3$

4.2 Calculated Asymptotic Variance of REML Estimator by EMM

Read solutions: Jiang (2002): p 79.

4.3 Tabulated Comparison

Review Jiang (2002): p 81.

5 General Asymptotic Covariance Matrix

Review textbook: p 41 and p 73.

6 References

◇ JiMing Jiang (2002), Empirical method of mements and its applications. Journal of Statistical Planning and Inference. 115: 69-84.