

Homework: 4-2 (Due: 11-27-2007)

In Binomial model ~~with~~ $Bi(n, p = \frac{e^\theta}{1+e^\theta})$

We have likelihood

$$\begin{aligned} f(y|n, \theta) &\propto \left(\frac{e^\theta}{1+e^\theta}\right)^y \left(\frac{1}{1+e^\theta}\right)^{n-y} \\ &= \frac{e^{\theta y}}{(1+e^\theta)^n} = e^{\theta y - n \log(1+e^\theta)} \end{aligned}$$

Please prove:

$$\int_{-\infty}^{+\infty} e^{\theta y - n \log(1+e^\theta)} d\theta < +\infty \quad \text{if } 1 \leq y \leq n-1.$$

Hint: $\log(1+e^x) \geq \max(0, x)$, work on Part $[\theta > 0]$ and $[\theta \leq 0]$ separately.